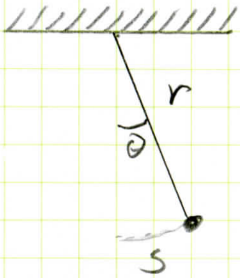
TMS PR 3.12

FOR A SIMPLE PENDULUM IN SMALL OSCILLATIONS ($\sin\theta \approx \theta$), SHOW

$$\omega_N = \sqrt{\frac{g}{r}}.$$

DISCUSS THE MOTION WHEN $F_{DRAG} = 2m\sqrt{gr}\,\dot{\theta}$.



THE MOTION & RESTORING FORCE ARE ALONG THE ARC, S

$$\sum F_s = m\ddot{s}$$

$$-mg\sin\theta = m\ddot{s}$$

$$\ddot{s} + g\sin\theta = 0$$

$s = r\theta$

$\dot{s} = r\dot{\theta}$

$\ddot{s} = r\ddot{\theta}$

CONVERTING TO POLAR COORDINATES

$$r\ddot{\theta} + g\sin\theta = 0$$

$$\ddot{\theta} + \frac{g}{r}\sin\theta = 0$$

TAKING SMALL OSCILLATIONS $\Rightarrow \sin\theta = \theta$

$$\Rightarrow \ddot{\theta} + \frac{g}{r}\theta = 0$$

THUS $\omega_N^2 = \frac{g}{r} \Rightarrow \boxed{\omega_N = \sqrt{\frac{g}{r}}}$

FOR $F_D = 2m\sqrt{gr}\,\dot{\theta}$ NSL BECOMES ($\sin\theta = \theta$)

$$-mg\theta - 2m\sqrt{gr}\,\dot{\theta} = mr\ddot{\theta}$$

$$\ddot{\theta} + 2\underbrace{\sqrt{\frac{g}{r}}}_{\beta = \sqrt{\frac{g}{r}}}\dot{\theta} + \underbrace{\frac{g}{r}}_{\omega_N = \sqrt{\frac{g}{r}}}\theta = 0$$

SINCE $\omega_N = \beta \Rightarrow$ CRITICAL DAMPING

THUS

$$\boxed{\theta(t) = (1+\beta t)Ae^{-\beta t} = \left(1+\sqrt{\frac{g}{r}}\,t\right)Ae^{-\sqrt{\frac{g}{r}}\,t}}$$